AP Rev. #10 Key

1) a) $x'(t) = e^{-t}\cos t - e^{-t}\sin t$

$x'(t) = e^{-t}(\cos t - \sin t)$

$e^{-t} \neq 0 \qquad \cos t = \sin t$

$t = \frac{\pi}{4}, \frac{5\pi}{4}$

Farthest left @ minimum

x': 0, π, 2π

$\oplus$ $\frac{\pi}{4}$ $\ominus$ $\frac{5\pi}{4}$ $\oplus$

$\searrow \nearrow$

$\boxed{t = \frac{5\pi}{4}}$

b) $x''(t) = \cancel{e^{-t}(-\sin t)} - e^{-t}\cos t - e^{-t}\cos t \cancel{+ e^{-t}\sin t}$

$x''(t) = -2e^{-t}\cos t$

$A(x''(t)) \quad + \quad x'(t) \quad + \quad x(t) \quad = 0$

$-2Ae^{-t}\cos t + e^{-t}\cos t - \cancel{e^{-t}\sin t} + \cancel{e^{-t}\sin t} = 0$

$e^{-t}(-2A + 1) = 0$

$e^{-t} \neq 0 \qquad -2A + 1 = 0$

$\boxed{A = \frac{1}{2}}$

2 a) Using tangent line approx @ $t=5$

$(5, 30)$ slope: $r'(5) = 2$ ft/min

$r - 30 = 2(t - 5)$

$r = 2t + 20$

$r = 2(5.4) + 20$

$\boxed{r = 30.8}$ ft @ $t = 5.4$

Estimate is greater than actual b/c $r'(t)$ is decreasing on $5 < t < 7$ (from table)

b) $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \qquad \frac{dr}{dt} = 2$ (from table) $\quad t=5, r=30$

$\frac{dV}{dt} = 4\pi(30)^2(2)$

$\frac{dV}{dt} = 7200\pi$ ft^3/min

3 a) $f'(x) = \frac{k}{2\sqrt{x}} - \frac{1}{x}$ $\qquad f''(x) = -\frac{k}{4x^{3/2}} + \frac{1}{x^2}$

b) $0 = \frac{k}{2\sqrt{x}} - \frac{1}{x}$

$\frac{1}{x} = \frac{k}{2\sqrt{x}}$

$k = \frac{2\sqrt{x}}{x}$

when $x=1$, $\boxed{k=2}$

$f''(x) = -\frac{2}{4x^{3/2}} + \frac{1}{x^2}$

$f''(x) = -\frac{1}{2x^{3/2}} + \frac{1}{x^2}$

$f''(1) = -\frac{1}{2} + 1$

= positive confirms concave up

$\therefore$ relative min @ $x=1$

c) PI on x-axis:

PI $\rightarrow$ $0 = -\frac{k}{4x^{3/2}} + \frac{1}{x^2}$

$\frac{k}{4x^{3/2}} = \frac{1}{x^2}$

$k = \frac{1}{x^2} \cdot 4x^{3/2}$

$k = \frac{4}{\sqrt{x}}$

$k = \frac{4}{\sqrt{e^4}}$

$k = \boxed{\frac{4}{e^2}}$

x-int from original eq:

$0 = k\sqrt{x} - \ln x$

$\ln x = k\sqrt{x}$

$k = \frac{\ln x}{\sqrt{x}}$

set =

$\frac{4}{\sqrt{x}} = \frac{\ln x}{\sqrt{x}}$

$4 = \ln x$

$x = e^4$

$k = \frac{\ln e^4}{\sqrt{e^4}}$

$= \frac{4}{(e^4)^{1/2}}$

$k = \boxed{\frac{4}{e^2}}$

4) a) PI where g' goes from incr to decr. or decr to incr.
@ $x = 1, 4$ (from looking at the graph

b) ab. max @ endpts or where g' goes from + to −
check: $x = -3, 2, 7$

from $x=-3$ to $x=2$, g' is more negative than positive which means g will decrease more than increase so g will be higher at $x=-3$ than at $x=2$. By same reasoning, g is higher @ $x=2$ than @ $x=7$
So absolute Max @ $x=-3$

5) a) $2x + 2 + 4y^3 \frac{dy}{dx} + 4\frac{dy}{dx} = 0$

$$\frac{\frac{dy}{dx}(4y^3 + 4)}{4(y^3+1)} = -\frac{\cancel{2}(x+1)}{{}_2 4(y^3+1)}$$

$$\frac{dy}{dx} = -\frac{(x+1)}{2(y^3+1)}$$

b) $m = \frac{-(-2+1)}{2(1+1)} = \frac{1}{4} \longrightarrow y - 1 = \frac{1}{4}(x+2)$

c) vertical tangent where $\frac{dy}{dx}$ is undef:

$y^3 + 1 = 0$

$y = -1 \rightarrow$ into original: $x^2 + 2x + (-1)^4 + 4(-1) = 5$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4, 2$

$\boxed{(-4,-1) \text{ and } (2,-1)}$

d) horiz. tangent where

$\frac{dy}{dx} = 0 \rightarrow x + 1 = 0$

$x = -1$

is there an x-int @ $(-1,0)$?

$(-1)^2 + 2(-1) + 0^4 + 0 \stackrel{?}{=} 5$

$1 - 2 \neq 5$ $\therefore$ No horiz. tangent @ an x-int.

6) $L'(5.5) \approx \frac{150-126}{7-4} = \frac{24}{3} = 8$ people/hr.